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Semiring satisfying the identity a.b = a + b + 1

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Abstract: Author determine some different additive and multiplicative structures of semiring which satisfies the identity a.b = a + b + 1. The motivation to prove this theorems in this paper is due to the results of P. Sreenivasulu Reddy and Guesh Yfter [6].

Keywords: Semiring, Rugular Semigroup

I. INTRODUCTION

Various concepts of regularity have been investigated by R.Croisot[7] and his study has been presented in the book of A.H.Clifford and G.B.Preston[1] as croisot's theory. One of the central places in this theory is held by left regularity. Bogdanovic and Ciric in their paper, "A note on left regular semigroups" considered some aspects of decomposition of left regular semigroups. In 1998, left regular ordered semigroups and left regular partially ordered Γ-semigroups were studied by Lee and Jung, kwan and Lee. In 2005, Mitrovic gave a characterization determining when every regular element of a semigroup is left regular. It is observed that many researchers studied on different structures of semigroups. The idea of generalization of a commutative semigroup was first introduced by Kazim and Naseeruddin in 1972 [2]. They named it as a left almost semigroup(LA-semigroup). It is also called an Abel Grassmanns groupoid (AG-semigroup)[2]. Q. Mushtaq and M. Khan[8] studied left almost semigroups in ideals, ideals in intra-regular left almost semigroups and proved that if P is left ideal and Q is right ideal of the regular left almost semigroup then PQ = P∩Q. They also proved that every right(left) ideal of regular LA-semigroup is an ideal and if S is an LA-semigroup having the left identity then S is total i.e.,S^2 = S. An LA- semigroup S is called regular if for each a∈S, there exist x∈S such that a = (ax)a. In the paper P. Sreenivasulu Reddy and Guesh Yfter [6] discussed some results on simple semirings.

Definition1.1. A semiring is a non empty set S on which operations of addition "+" and multiplication "." have been defined such that the following conditions are satisfied:
(i) (S, +) is a semigroup
(ii) (S, .) is a semigroup
(iii) Multiplication distributes over addition from either side.

Definition1.2. An element a of a semigroup (S, .) is said to be regular if there exist x in S such that (ax)a = a
Definition1.3. A semigroup (S, .) is called regular if every element of S is regular.

Examples of regular semigroups:
i) Every group is regular.
ii) Every inverse semigroup is regular.
iii) The bicyclic semigroup is regular.
iv) Any full transformation semigroup is regular.
v) A Rees matrix semigroup is regular.

Definition1.4. A semigroup S is called quasi- seperative if for any x,y∈ S, $x^2 = xy = y^2$ implies x = y.

Definition1.5. A semigroup S is called weakly seperative if $x^2 = xy = yx = y^2$ implies x = y for all x,y in S.
Definition1.6. A semigroup S is called seperative if i) $x^2 = xy$ and $y^2 = yx$ then x = y
ii) $x^2 = yx$ and $y^2 = xy$ then x = y

II. PRELIMANARIES

Theorem 2.1. Let (S, +, .) be a left almost semiring and (S, .) is a regular semigroup which satisfies the identity a.b = a + b + 1, for all a, b∈(S, .) then (S, +) is a i) regular semigroup
ii) left (right) regular semigroup iii) completely regular semigroup

Proof. Let (S, +, .) be a left almost semiring and (S, .) be a regular semigroup and it satisfiesthe identity a.b = a + b + 1, a, b∈(S, .) and 1 is a multiplicative identity in (S, .)

i) Since (S, .) is a regular semigroup a = axa =(ax)a = (a + x + 1).a = a + x + 1 + a + 1= a + x + 1.a = a + x + a. Hence a is a regular element in S. Therefore (S, +) is a regular semigroup.

ii) Let a + a + x = a + a + x.1 = a + a + x + 1 + 1= a + a.x + 1= a + ax + 1= aax = a. Hence a is a left regular element of (S, +). Therefore, (S, +) is a left regular semigroup
Similarly (S, +) is also right regular semigroup.

iii) Since (S, .) and (S, +) are regular and left regular , a + x + a = a = a + a + x = x + a + a. So, a + x + a = a + a + x → x + a + x + a = x + a + a + x → x + a = a + x. Hence, (S, +) is a completely regular semigroup.

Theorem 2.2. Let (S, +, .) be a left almost semiring and (S, +) is a regular semigroup which satisfies the identity a.b = a + b + 1, For all a, b ϵ (S, +) then (S, .) is i) regular semigroup
ii) left (right) regular semigroup iii) completely regular semigroup

Proof. Let (S, +, .) be a left almost semiring and (S, +) be a regular semigroup and it satisfies the identity a.b= a + b + 1 a, bϵ(S, +) and 1 is multiplicative identity in (S, .)
Since (S, +) is a regular semigroup, a = a + x + a = a + x + a.1 = a + x + a + 1 + 1 = a + x.a +1 = a + xa + 1 = a + ax + 1 = aax = a^2x.
a is a left regular element of (S, .). Hence every element of (S, +) is a left regular element of (S, .). Therefore (S, .) is a left regular semigroup.
Similarly, (S, .) is also a right regular semigroup.

ii) Since (S, +) is a regular semigroup, a = a + x + a → a = a + x + a.1 = x + a + 1 +1 = a + x.a + 1 = a + xa + 1 = axa. Hence, a is a regular element of (S, ·). Therefore, (S, ·) is a regular semigroup.
iii) Since (S, ·) is a regular and left(right) regular semigroup, axa = xa^2 → axax = xa^2x → ax = xa. Therefore, (S, ·) is a completely regular semigroup.

Theorem 2.3. Let (S, +, ·) be a semiring and (S, ·) is a seperative semigroup which satisfies the identity a.b = a + b + 1 for all a, bϵ(S, .) then (S, +) be a i) quasi-seperative semigroup ii) weakly seperative semigroup iii) seperative semigroup.

Proof. Let (S, +, ·) be a semiring and (S, ·) is a seperative semigroup which satisfies the identity a.b = a + b + 1 for all a, bϵ(S, .) and 1 is the multiplicative identity.
Consider a + a = a + b ⇒ a + a + 1 = a + b +1 ⇒ a.a = a.b ⇒ a^2 = ab (since (S, ·) is quasi-seperative)
Similarlly, a + b = b + b ⇒ a + b + 1 = b + b + 1 ⇒ a.b = b.b ⇒ ab = b^2 ⇒ a = b.
Hence, a + a = a + b = b + b ⇒ a = b. Therefore, (S, +) is a quasi-seperative.
ii) Let a + b = a.1 + b = a + 1 + 1 + b = a + b + 1 + 1 = 1+ b + 1 + a = 1.b + a = b + a
Hence, (S, +) is a commutative semigroup.
Since (S, +) is quasi- seperative and commutative semigroup we have
a+ a = a + b = b + a = b + b ⇒ a = b. Therefore, (S, +) is waekly seperative. iii)
Since (S, +) is quasi-seperative and weakly- seperative, (S, +) is seperative.

Theorem 2.4. Let (S, +, ·) be a semiring and (S, +) is a quasi-seperative semigroup which satisfies the identity a.b = a + b + 1 then (S, ·) is a i) quasi-seperarive semigroup ii) weakly- seperative semigroup iii) seperative semigroup.

Proof. Proof of the theorem is similar to the above proof.

Theorem 2.5. Let (S, +, ·) be a left almost semiring which satisfies the identity a.b = a + b + 1 for all a,bϵ(S, +, .) then (S, +, ·) is a i) commutative ii) medial iii) permutable.

Proof. Let (S, +, ·) be a left almost semiring which satisfies the identity a.b = a + b + 1 for all a, bϵ (S, +, .). Consider, a + b = a.1 + b = a + 1 + 1 + b = 1 + 1 + a + b = b + 1 + a + 1 = b + 1.a = b + a ⇒ a + b = b + a. Hence (S, +) is a commutative.
Again, a.b = a + b + 1 = a + b.1 + 1 = a + b + 1 + 1 +1 = a + 1 + 1 + b + 1 = b + 1 + 1 + a + 1 = b + a + 1 + 1 + 1 = b + a + 1.1 = b + a + 1 ⇒ a.b =b.a Hence (S, ·) is commutative. Therefore, (S, +, ·) is a commutative LA-semiring.
Since (S, +, ·) is a commutative it is easy to prove (S, +, ·) is medial and permutable.

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